Order of Operations

1. Perform any operations inside grouping symbols.
2. Simplify any term with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

EXAMPLE 1: Simplify

$2^2 \div 2 \times (9-7) + 8$ **Subtract inside the grouping symbols.**

$2^2 \div 2 \times (2) + 8$ **Simplify exponent.**

$4 \div 2 \times 2 + 8$ **Do multiplication/division in order from left to right.**

$2 \times 2 + 8$ **Do multiplication/division in order from left to right.**

$4 + 8$ **Add.**

12 **The answer is 12.**

DIRECTIONS: No Calculators. Simplify each expression. Evaluate if necessary. Show Work!

1) $2.7 + 3.6 \times 4.5$	2) $3[4(8-2)+5]$	3) $4 + 3(15 - 2^3)$

Distributive Property

Distributive Property of Addition	$a(b + c) = a \cdot b + a \cdot c$
Distributive Property of Subtraction	$a(b - c) = a \cdot b - a \cdot c$

*** Draw arrows to from the term outside the parentheses to the terms inside to show that the term outside is distributed to each term inside.**

EXAMPLE 1: Simplify by using the Distributive Property

$3(2x + 3)$

$3(2x + 3)$ **Draw arrows.**

$3(2x) + 3(3)$ **Use the Distributive Property.**

$6x + 9$ **Simplify.**

EXAMPLE 1: Simplify by using the Distributive Property

$-(4x + 7)$

$-1(4x + 7)$ **Rewrite using the Multiplication Property of -1**

$-1(4x + 7)$ **Draw arrows.**

$-1(4x) + (-1)(7)$ **Use the Distributive Property**

$-4x - 7$ **Simplify.**

DIRECTIONS: Show Work!... Simplify by using the Distributive Property.

1) $2(5x + 4)$	2) $\frac{1}{4}(12x - 8)$	3) $4(7x - 3)$
4) $-5(4 + 2x)$	5) $6(5 - 3x)$	6) $0.1(30x - 50)$
7) $-\frac{2}{3}(2x - 4)$	8) $(3x + 4)7$	9) $8(x + y)$
10) $-(4x + 3)$	11. $-(-2x + 1)$	12. $-(-6x - 3)$
13. $\frac{2}{5}(5k + 35) - 8$	14. $-4x + 3(2x - 5)$	15. $\frac{3}{4}(8x - 10) - 2(x - 15)$

Operations with Rational Numbers - Subtraction

To subtract a number, first rewrite the problem as an addition sentence by adding the opposite of the second number. Then, follow the rules for addition of numbers.

EXAMPLES:

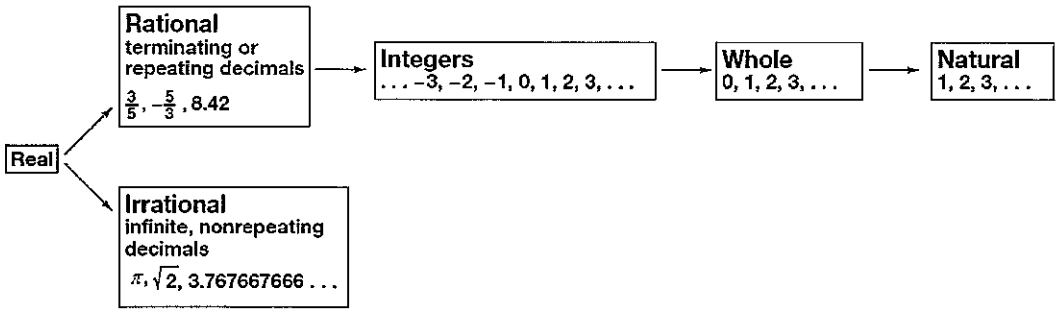
$6 - 2$ $6 + -2$ **Rewrite to add the opposite of the number.** **4** **Follow rules for addition.**	$-6 - 2$ $-6 + -2$ **Rewrite to add the opposite of the number.** **-8** **Follow rules for addition.**
$-6 - (-2)$ $-6 + 2$ **Rewrite to add the opposite of the number.** **-4** **Follow rules for addition.**	$6 - (-2)$ $6 + 2$ **Rewrite to add the opposite of the number.** **8** **Follow rules for addition.**

DIRECTIONS: No Calculators. Simplify. Evaluate when necessary. Show Work!

1) $7 - 12$	2) $6 - 9$	3) $4 - (-5)$
4) $7 - (-3)$	5) $-3.1 - (-5.4)$	6) $8.3 - 5.1$
7) $-7.8 - 6.6$	8) $-4.8 - 2.5$	9) $a - b$ for $a = -4$ and $b = 3$
10) $-a - b$ for $a = -4$ and $b = 3$	11) $a - (-b)$ for $a = -4$ and $b = 3$	12) $-a - (-b)$ for $a = -4$ and $b = 3$

Exploring Real Numbers

The chart below shows the different classifications of real numbers:



EXAMPLES:

Given the numbers $-4.4, \frac{14}{5}, 0, -9, 1\frac{1}{4}, -\pi$ and 32, tell which numbers belong to each set.

Natural:	32	numbers used to count
Whole:	0, 32	natural numbers and zero
Integers:	0, −9, 32	whole numbers and their opposites
Rational:	$-4.4, \frac{14}{5}, 0, -9, 1\frac{1}{4}, 32$	integers and terminating and nonrepeating decimals
Irrational:	$-\pi$	infinite, nonrepeating decimals
Real:	$-4.4, \frac{14}{5}, 0, -9, 1\frac{1}{4}, -\pi, 32$	rational and irrational numbers

DIRECTIONS: No Calculators. Name the set(s) of numbers to which each number belongs.

1. -29
2. 7.8
3. 0.384
4. 14.8888.....
5. $0.\overline{57}$
6. $12\frac{4}{5}$
7. $-3\frac{1}{9}$
8. $\sqrt{49}$
9. $\sqrt{10}$
10. $\sqrt{\frac{1}{4}}$

Simplifying Radical Expressions

A radical expression is in **simplest form** when the radicand contains no perfect square factors and the denominator, if applicable, does not contain a radical.

EXAMPLES:

Condition	Not in Simplest Form	How to Simplify	Simplest Form
The Multiplication Property of Square Roots is used to simplify the radical.			
The expression under the radical sign has no perfect square factors other than 1.	$\sqrt{20}$	Rewrite as a product of perfect squares and other factors. $= \sqrt{4 \cdot 5}$ $= \sqrt{4} \cdot \sqrt{5}$	$2\sqrt{5}$
The Division Property of Square Roots is used to simplify the radical.			
The expression under the radical sign is a fraction.	$\sqrt{\frac{16}{25}}$	Separate into two radical expressions. Simplify each separately. $\frac{\sqrt{16}}{\sqrt{25}}$	$\frac{4}{5}$
The denominator contains a radical expression that is not a perfect square	$\frac{3}{\sqrt{2}}$	Rationalize the denominator by multiplying the fraction by a radical expression equal to 1. $= \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$	$\frac{3\sqrt{2}}{2}$

DIRECTIONS: No Calculators. Simplify each radical expression.

1) $\sqrt{22} \cdot \sqrt{8}$	2) $\sqrt{147}$	3) $\sqrt{160}$
4) $\frac{\sqrt{96}}{\sqrt{12}}$	5) $\sqrt{\frac{17}{144}}$	6) $\sqrt{18} \cdot \sqrt{8}$

Two-Step Equations

To isolate the variable,
1. First, eliminate addition or subtraction by using inverse operations.
2. Then, eliminate addition or subtraction by using inverse operations.

EXAMPLE 1:

$3x + 4 = 10$

$-4 \quad -4$ *Subtraction Property of Equality*

$3x = 6$

$\frac{3x}{3} = \frac{6}{3}$ *Division Property of Equality*

$x = 2$ *Solution*

CHECK:

$3x + 4 = 10$ *Recopy the original equation*

$3(2) + 4 = 10$ *Substitute the solution for the variable*

$6 + 4 = 10$ *Use Order of Operation to simplify*

$10 = 10$ *Make sure values match*

EXAMPLE 2:

$1 = -\frac{-k}{5} - 3$

$+3 \quad +3$ *Addition Property of Equality*

$4 = -\frac{-k}{5}$

$4(5) = -\frac{-k}{5}(5)$ *Multiplication Property of Equality*

$20 = -(-k)$

$20 = k$

CHECK:

$1 = -\frac{-k}{5} - 3$ *Recopy the original equation*

$1 = -\frac{-20}{5} - 3$ *Substitute the solution for the variable*

$1 = -(-4) - 3$ *Use Order of Operation to simplify*

$1 = 4 - 3$

$1 = 1$ *Make sure values match*

DIRECTIONS: No Calculators. Solve each equation. Check your solution.

1) $5a + 2 = 7$	2) $3x - 7 = 35$	2) $67 = -3y + 16$
#1 CHECK	#2 CHECK	#3 CHECK
4) $4s - 13 = 51$	5) $5.8n + 3.7 = 29.8$	6) $11.6 + 3a = -16.9$

Multi-Step Equations

To isolate the variable,

1. Distribute if necessary.Combine like terms on one side of the equation, if necessary. [*This should look like a two-step equation now!*]
2. Eliminate addition or subtraction by using inverse operations.
3. Eliminate multiplication or division by using inverse operations.

EXAMPLE 1:

$3c - 8c + 7 = -18$

$-5c + 7 = -18$ *Combine like terms*

$-7 \quad -7$ *Subtraction Property of Equality*

$-5c = -25$

$\frac{-5c}{-5} = \frac{25}{-5}$ *Division Property of Equality*

$C = 5$

EXAMPLE 2:

$-4d + 2(3 + d) = -14$

$-4d + 2 \cdot 3 + 2 \cdot d = -14$ *Use the Distributive Property*

$-4d + 6 + 2d = -14$

$-2d + 6 = -14$ Combine like terms

$-6 \quad -6$ Subtraction property of equality

$-2d = -20$

$\frac{-2d}{-2} = \frac{-20}{-2}$ *Division Property of Equality*

$d = 10$ *Solution*

CHECK:

$3c - 8c + 7 = -18$ *Recopy the original equation*

$3(5) - 8(5) + 7 = -18$ *Substitute solution for the variable*

$15 - 40 + 7 = -18$ *Use Order of Operation to simplify*

$-25 + 7 = -18$

$-18 = -18$ *Make sure values match*

CHECK:

$-4d + 2(3 + d) = -14$ *Recopy the original equation*

$-4(10) + 2(3 + (10)) = -14$ *Substitute solution for the variable*

$-40 + 2(13) = -14$ *Use Order of Operation to simplify*

$-40 + 26 = -14$

$-14 = -14$ *Make sure values match*

Directions: No Calculators. Solve each equation. Check your solution.

1) $2n + 3n + 7 = -41$	2) $2b - 6 + 3b = 14$	3) $3(t - 12) = 27$
#1 CHECK	#2 CHECK	#3 CHECK

Equations with Variables on Both Sides

To isolate the variable,

1. Distribute if necessary.
2. Combine like terms on *each* side of the equation, if necessary.
3. Get the variable on one side of the equation by using inverse operations (either addition or subtraction). [*The equation should look like a two-step equation now!*]
4. Eliminate addition or subtraction by using inverse operations. Eliminate multiplication or division by using inverse operations.

EXAMPLE: Solve and check.

$3k + 5 = 4(k + 1)$

$3k + 5 = 4k + 4$ *Use the Distributive Property*

$-4k \quad -4k$ *Subtraction Property of Equality**

$-k + 5 = 4$

$-5 \quad -5$ Subtraction property of Equality

$-k = -1$

$k = 1$

CHECK:

$3k + 5 = 4(k + 1)$ *Recopy the original equation*

$3(1) + 5 = 4((1) + 1)$ *Substitute solution for the variable*

$3 + 5 = 4(2)$

$8 = 8$

** Another option is to subtract 3k from both sides. The resulting equivalent equation would be $5 = k + 4$. Both equations will lead to the same solution.*

DIRECTIONS: No Calculators. Solve and graph each inequality. Include a check.

1) $7 - 2n = n - 14$	2) $3d + 8 = 2d - 7$	3) $2(6 - 4d) = 25 - 9d$
#1 CHECK	#2 CHECK	#3 CHECK

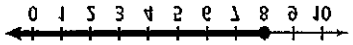
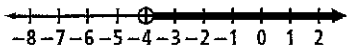
One-Step Inequalities – Addition and Subtraction

To solve an inequality with addition and subtraction, follow the same procedure for equations to isolatethe variable. There are infinitely many solutions to an inequality. To represent the solution set, the solution to the inequality is graphed on a number line.

For $\leq$ and $\geq$, a **closed** circle is placed on the boundary value since the value **is** part of the solution set. For $<$, $>$, and $\neq$ an **open** circle is placed on the boundary value since the value **is not** part of the solution set.

To represent the remainder of the solution set, shade the segment of the number line that satisfies the values of the solution set.

To check the solution set, selectis shaded in the graph). Substitute to ensure the final inequality statement is true. a value that satisfies the inequality (this will be a number that

EXAMPLE 1: Solve $y - 10 \leq -2$ $+10 \quad +10$ *Addition Property of Inequality* $y \leq 8$ *Solution* ** Any value that is less than or equal to 8 is part of the solution set.*	GRAPH and CHECK Example #1  $y - 10 \leq -2$ *Recopy the original inequality* $7 - 10 \leq -2$ *Substitute a value from the solution set** $-3 \leq -2$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used to check the solution.*
EXAMPLE 2: Solve $12 < w + 16$ $-16 \quad -16$ *Subtraction Property of Inequality* $-4 < w$ *Solution* $w > -4$ ***An equivalent inequality where the variable is on the right can also be a representation of the solution set. Notice that the inequality symbol must flip!* ** Any value that is less than or equal to -4 is part of the solution set.*	GRAPH and CHECK Example #2  $12 < w + 16$ *Recopy the original inequality* $12 < -3 + 16$ *Substitute a value from the solution set** $12 < 13$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used to check the solution.*

DIRECTIONS: No Calculators. Solve and graph each inequality. Include a check.

1) $n - 7 \geq 2$	2) $x + 1 \leq -3$	3) $d - 13 \leq -8$

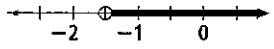
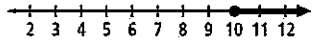
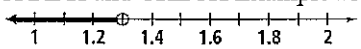
One-Step Inequalities – Multiplication and Division

To solve an inequality with multiplication and division, follow the same procedure for equations to isolate the variable.

There are infinitely many solutions to an inequality.

When you multiply or divide an inequality by a **negative** number, you must ***flip the inequality symbol.***

To represent the solution set, follow the same procedure explained in the previous lesson to graph the solution set on a number line.

EXAMPLE 1: Solve $\frac{b}{2} > -\frac{3}{4}$ $2 * \frac{b}{2} > -\frac{3}{4} * 2$ *Multiplication Property of Inequality* $b > -1.5$ *Solution* ** Any value that is greater than -1.5 is part of the solution set.*	GRAPH and CHECK Example #1  $\frac{b}{2} > -\frac{3}{4}$ *Recopy the original inequality* $\frac{-1}{2} > -\frac{3}{4}$ *Substitute a value from the solution set** $-\frac{1}{2} > -\frac{3}{4}$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used to check the solution.*
EXAMPLE 2: Solve $-100c \leq -1,000$ $\frac{-100c}{-100} \leq \frac{-1,000}{-100}$ *Division Property of Inequality* $p \geq 10$ *Solution* *** Notice that the inequality symbol must* ***flip*** *because you divide both sides of the inequality by* ***negative*** *number to isolate the variable!* ** Any value that is greater than or equal to 10 is part of the solution set.*	GRAPH and CHECK Example #2  $-100c \leq -1,000$ *Recopy the original inequality* $-100(10) \leq -1,000$ *Substitute a value from the solution set** $-1,000 \leq -1,000$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used for to check the solution.*
EXAMPLE 3: Solve $\frac{3g}{3} < \frac{3.9}{3}$ $g < 1.3$ *Solution* ** Any value that is less than 1.3 is part of the solution set.* *Note: There is no need to flip the inequality symbol because you are dividing both sides by positive 3.*	GRAPH and CHECK Example #3  $3g < 3.9$ *Recopy the original inequality* $3(1) < 3.9$ *Substitute a value from the solution set** $3 < 3.9$ *Make sure the inequality statement is true* ** Any value that is "shaded in" on the graph could be used to check the solution.*